

Lifshitz Solitons

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Abstract

We numerically obtain a class of soliton solutions for Einstein gravity in $(n+1)$ dimensions coupled to massive abelian gauge fields and with a negative cosmological constant with Lifshitz asymptotic behaviour. We find that for all $n \geq 3$, a discrete set of magic values for the charge density at the origin (guaranteeing an asymptotically Lifshitz geometry) exists when the critical exponent associated with the Lifshitz scaling is $z = 2$; moreover, in all cases, a single magic value is obtained for essentially every $1 < z < 2$, yet none when $z > 2$ sufficiently.

1 Introduction

Since its proposal by Maldacena [1], the AdS/CFT correspondence has proven to be of appreciably broad theoretical utility, providing new lines of research into both quantum gravity and quantum chromodynamics. It conjectures the existence of a holographic duality between strongly interacting field theories and weakly coupled gravitational dynamics in an asymptotically AdS bulk spacetime of one dimension greater.

This idea has been extended in recent years beyond high energy physics to describe strongly coupled systems in condensed matter physics. In particular, it has enjoyed useful applicability to theories that model quantum critical behaviour [2, 3, 4, 5] characterized by Lifshitz scaling – that is, a scaling transformation of the form

$$t \rightarrow \lambda^z t, \quad r \rightarrow \lambda^{-1} r, \quad \mathbf{x} \rightarrow \lambda \mathbf{x}, \quad (1)$$

where $z \geq 1$ is a dynamical critical exponent representing the degree of anisotropy between space and time. For instance, when $z = 2$, the scaling symmetry given by (1) is associated with a $(2+1)$ -dimensional field theory of strongly correlated electron systems.

Such theories are conjectured [6] to be holographically dual to gravitational theories whose solutions are asymptotic to the so-called Lifshitz metric,

$$ds^2 = \ell^2 \left(-r^{2z} dt^2 + \frac{dr^2}{r^2} + r^2 d\mathbf{x}^2 \right), \quad (2)$$

where the coordinates (t, r, x^i) are dimensionless and the only length scale in the geometry is ℓ . Metrics asymptotic to (2) can be generated as solutions to the equations of motion that follow from the action

$$I = \frac{1}{16\pi} \int d^{n+1}x \sqrt{-g} \left(R - 2\Lambda - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} H_{\mu\nu} H^{\mu\nu} - \frac{C}{2} B_\mu B^\mu \right), \quad (3)$$

where Λ is the cosmological constant, $F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$ with A_μ representing the Maxwell gauge field, and $H_{\mu\nu} = \partial_{[\mu} B_{\nu]}$ is the field strength of the Proca field B_μ with mass $m^2 = C$.

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This dual theory, referred to as Lifshitz gravity, is known to describe neutral and charged black holes [7][8][9][10] whose metrics are asymptotic to the metric (2). It is possible to replace the Proca field with higher-order curvature terms [11][12][13] and attain black hole metrics with the same asymptotic structure.

Here we explore the possibility of obtaining soliton solutions to the field equations that follow from the action (3) when the Maxwell field is zero. Originally referred to as Lifshitz stars [7], these objects have non-singular spacetime geometries with the same asymptotics as their black hole counterparts. We prefer to call them solitons since, unlike stars, there is no sharp boundary between a vacuum and non-vacuum region. While black hole solutions in $(n+1)$ dimensions [10] and soliton solutions in $(2+1)$ [14] and $(3+1)$ dimensions [7] have already been found, solitons in higher dimensions have not been investigated thus far.

Here our central aim is to obtain soliton solutions for general (n, z) , and to discuss some of their consequences. For any dimension n , we find that a single soliton solution to the field equations exists for $z < 2$, whereas for $z > 2$ we are unable to obtain any solutions. For $z = 2$ we find a discrete set of soliton solutions in any dimensionality. Our results are numerical, and so we are able to obtain additional soliton solutions for $|z - 2| < \epsilon$ for sufficiently small ϵ , where ϵ decreases with increasing dimensionality.

The rest of this paper is organized as follows. In Section 2, we give the field equations for Lifshitz gravity and then reduce them to a system of four first-order ODEs. In Section 3, we numerically solve the equations of motion for $n = 3, 4, 5, 6$ and list the corresponding magic values for the central charge density in each case. Section 4 concludes the paper, and the Appendix gives the small-radius series coefficients of the metric and gauge functions (needed to determine a set of initial conditions for Section 3).

2 The $(n+1)$ -Dimensional Field Equations

Given the action (3) with $A_\mu = 0$, the variational principle yields the field equations [10]:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}, \quad (4)$$

$$\nabla^\mu H_{\mu\nu} = C B_\nu, \quad (5)$$

$$\partial_{[\mu} B_{\nu]} = H_{\mu\nu}, \quad (6)$$

where the energy-momentum tensor of the gauge fields is

$$T_{\mu\nu} = -\frac{1}{2} \left[\frac{1}{4} H_{\rho\sigma} H^{\rho\sigma} g_{\mu\nu} - H_\mu^\rho H_{\rho\nu} + C \left(\frac{1}{4} B_\rho B^\rho g_{\mu\nu} - B_\mu B_\nu \right) \right]. \quad (7)$$

The general $(n+1)$ -dimensional metric preserving the basic symmetries (1) can be written as

$$ds^2 = \ell^2 \left(-r^{2z} f^2(r) dt^2 + \frac{g^2(r) dr^2}{r^2} + r^2 d\Omega_k^2 \right), \quad (8)$$

where

$$d\Omega_k^2 = \begin{cases} d\theta_1^2 + \sum_{i=2}^{n-1} \prod_{j=1}^{i-1} \sin^2 \theta_j d\theta_i^2, & k = 1, \\ d\theta_1^2 + \sinh^2 \theta_1 \left(d\theta_2^2 + \sum_{i=3}^{n-1} \prod_{j=2}^{i-1} \sin^2 \theta_j d\theta_i^2 \right), & k = -1, \\ \sum_{i=1}^{n-1} d\theta_i^2, & k = 0 \end{cases} \quad (9)$$

is the metric of an $(n-1)$ -dimensional hypersurface with constant curvature $(n-1)(n-2)k$.

The Proca field is assumed to be

$$B_t = q\ell r^z f(r) j(r), \quad H_{tr} = q\ell z r^{z-1} g(r) h(r) f(r), \quad (10)$$

with all other components either vanishing or given by antisymmetrization. The asymptotic conditions $f(r)$, $g(r)$, $h(r)$, $j(r) \rightarrow 1$ as $r \rightarrow \infty$ (required to ensure that any solutions obtained are asymptotic to (2)) impose

the following constraints:

$$C = \frac{(n-1)z}{\ell^2}, \quad q^2 = \frac{2(z-1)}{z}, \quad \Lambda = -\frac{(z-1)^2 + n(z-2) + n^2}{2\ell^2}. \quad (11)$$

It can be shown [10] that the above reduce the field equations (4)-(6) to a system of four first-order ODEs,

$$r \frac{df}{dr} = \frac{f}{4(n-1)r^2} \left\{ 2 \left[(n-1)(z-1)j^2 - z(z-1)h^2 + (z-1)^2 + n(z-2) + n^2 \right] r^2 g^2 + 2(n-1) \left[(n-2)k\ell^2 g^2 - (n+2z-2)r^2 \right] \right\}, \quad (12)$$

$$r \frac{dg}{dr} = \frac{g}{4(n-1)r^2} \left\{ 2 \left[(n-1)(z-1)j^2 + z(z-1)h^2 - (z-1)^2 - n(z-2) - n^2 \right] r^2 g^2 - 2(n-1) \left[(n-2)k\ell^2 g^2 - nr^2 \right] \right\}, \quad (13)$$

$$r \frac{dj}{dr} = \frac{-j}{4(n-1)r^2} \left\{ 2 \left[(n-1)(z-1)j^2 - z(z-1)h^2 + (z-1)^2 + n(z-2) + n^2 \right] r^2 g^2 + 2(n-1) \left[(n-2)k\ell^2 g^2 - (n-2)r^2 \right] \right\} + zgh, \quad (14)$$

$$r \frac{dh}{dr} = (n-1)(jg - h). \quad (15)$$

which, in general, cannot be solved analytically.

3 Numerical Solutions for $(n+1)$ -Dimensional Solitons

Series solutions for the field equations at large r have been previously obtained [7][8][11].

Before we can numerically obtain soliton solutions to the equations of motion (12)-(15), we require a set of initial conditions i.e. the values of $f(\varepsilon)$, $g(\varepsilon)$, $j(\varepsilon)$ and $h(\varepsilon)$ for some $0 < \varepsilon \ll 1$. For soliton solutions we demand that the metric be regular and that the Proca field have vanishing field strength at the origin. To this end, consider the following series expansions for small r :

$$f(r) = \frac{1}{r^z} \sum_{p=0}^{\infty} f_p r^{2p}, \quad g(r) = r \sum_{p=0}^{\infty} g_p r^{2p}, \quad j(r) = \sum_{p=0}^{\infty} j_p r^{2p}, \quad h(r) = r \sum_{p=1}^{\infty} h_p r^{2p}. \quad (16)$$

It is possible to write all of the coefficients $\{f_p, g_p, j_p, h_p\}_{p=0}^{\infty}$ simply in terms of f_0 and j_0 (which is the Proca charge density at the center of the soliton). In the Appendix, we give the full expressions for the first three terms in the series for each function in (16). We find that only for $k=1$ are the series solutions finite and real, so we shall henceforth set $k=1$.

We can now numerically solve the system (12)-(15) using the shooting method, taking as initial conditions (17)-(20) (given in the Appendix) truncated after two terms, evaluated at $\varepsilon = 10^{-6}$. Furthermore, we rescale all quantities in units of ℓ , effectively setting $\ell = 1$.

As previously noted, (17)-(20) are determined solely by two parameters, f_0 and j_0 . The value of the former can be easily assigned. Since f_0 appears only as an overall factor in (17), the system (12)-(15) can be solved by setting $f_0 = 1$ as an initial condition. If $g(r)$, $h(r)$, $j(r) \rightarrow 1$ and $f(r) \rightarrow c \neq 1$ as $r \rightarrow \infty$, we simply change the initial value of $f(r)$, to $f_0 = 1/c$ and so obtain the desired asymptotic behaviour.

The determination of the latter, however, is not quite so trivial. We find in general that, as for the $(3+1)$ -dimensional case [7], the conditions $f(r)$, $g(r)$, $h(r)$, $j(r) \rightarrow 1$ as $r \rightarrow \infty$ can be satisfied only for certain discrete values of j_0 , known as ‘magic’ values [7]. These correspond to the intercepts of the function $j_0 \gamma_0$; here, $\gamma_0 := j(r_L) - 1$, where $j(r)$ is the numerical solution to (14) dependent upon our choice of j_0 , and r_L is picked to be very large. We furthermore expect these intercepts to coincide with those of the functions*

* Although these intercepts are, of course, the same as just those of α_0 , β_0 and γ_0 , multiplication by j_0 makes our plots more readable.

$j_0\alpha_0$ and $j_0\beta_0$; here, $\alpha_0 := g(r_L) - 1$ and $\beta_0 := h(r_L) - 1$, where $g(r)$ and $h(r)$ are the numerical solutions to (13) and (15) respectively, again depending upon the value of j_0 . For $z = 2$ in $(3 + 1)$ -dimensions these intercepts correspond to the removal of a zero mode in the large- r linearized field equations.

In all of our work, we use $r_L = 10^5$ unless otherwise stated. Thus, plotting $j_0\gamma_0$ (as a function of j_0) for different n , and different z for each n , will suffice to give us the magic values, if any exist.

We will consider three cases: $1 < z < 2$, $z = 2$ and $z > 2$.

3.1 Case I: $1 < z < 2$

We find that for all $1 < z < 2$ and any $n \geq 3$, a single magic value of j_0 exists. Concordantly, here we essentially always observe a single intercept of the function $j_0\gamma_0$. The only possible exception is for $|z - 2|$ *small* (generally by less than 0.1) in which case we found numerically that more than one magic value may exist. Such scenarios are qualitatively comparable to those where z is exactly 2. Since we discuss this case at length in the next subsection we shall not elaborate on the small $|z - 2|$ cases any further here.

Figure 1 shows this for $n = 3$ and $n = 4$, with the situation being qualitatively very similar for all other n . Moreover, in Figure 2 we give the magic values that we have computed for $n = 3, 4, 5, 6$ as a function of $1 < z < 2$. We see that (in any dimension) the magic value increases with increasing z .

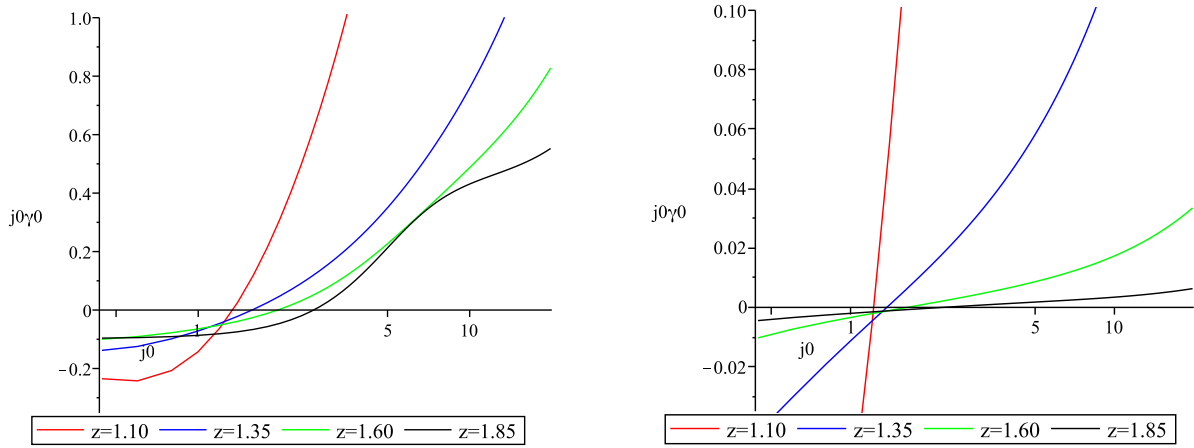


Figure 1: Plots of $j_0\gamma_0$ as a function of j_0 for different values of $1 < z < 2$. (The intercepts are the magic values corresponding to each case.) Left: $n = 3$. Right: $n = 4$.

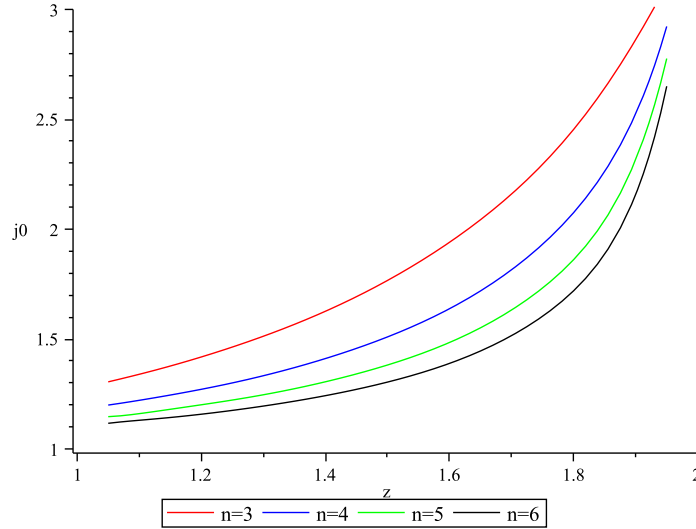


Figure 2: Magic values of j_0 as a function of $1 < z < 2$ for $n = 3, 4, 5, 6$. This plot was obtained by choosing, for each n , a discrete set of nine values of z equally spaced along the given interval, numerically computing the magic values for each (i.e. the intercepts corresponding to the kinds of graphs depicted in Figure 1), and then polynomially interpolating between them.

3.2 Case II: $z = 2$

In this case, a set of magic values exist for any n for all cases we numerically investigated. Figure 3a depicts the functions $j_0\alpha_0$, $j_0\beta_0$ and $j_0\gamma_0$ for $n = 4, 5$ while $j_0\gamma_0$ for $n = 6$ is plotted Figure 3b. We find that the larger the value of n , the more rapidly oscillating (and hence numerically unstable) these functions become – and also, the more difficult we find it to obtain convergence of their intercepts with satisfactory accuracy.

Figure 4 lists the numerical values thereof for $n = 3, 4, 5, 6$, i.e. the magic values in each dimension. For the plot of $j_0\gamma_0$ corresponding to $n = 3$, see [7]; note that in this case, we recover the same magic values as those obtained therein.

Furthermore, we plot the metric and gauge functions corresponding to the lowest magic value for $n = 4, 5, 6$ in Figure 5a. Accordingly, all of these are observed to converge to 1 as $r \rightarrow \infty$ and, as generally expected for solitons, we see that $g(r)$ and $h(r)$ vanish as $r \rightarrow 0$. In addition, the function $r^z f(r)$, again for different n , is plotted in Figure 5b.

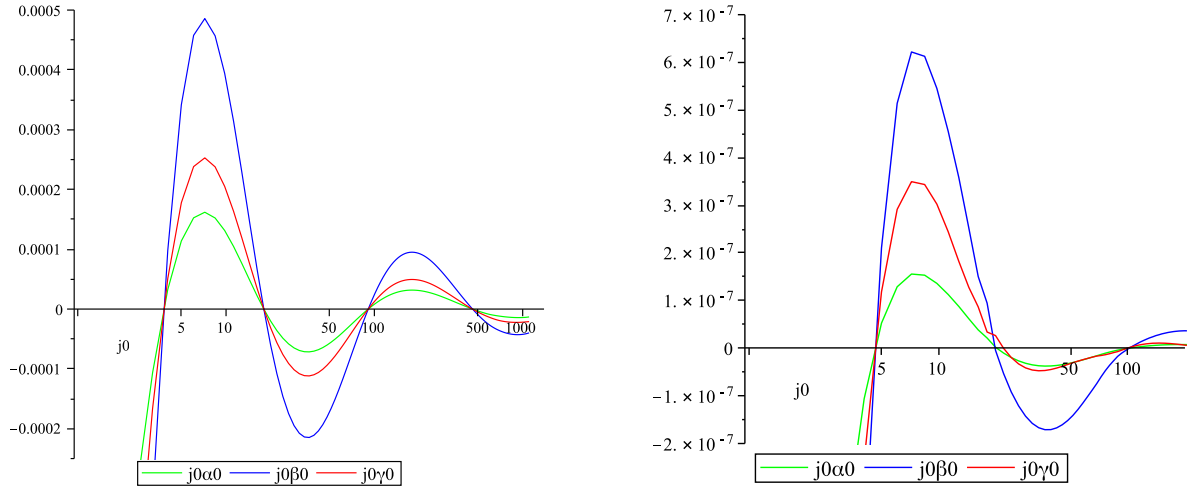


Figure 3a: Plots of $j_0\alpha_0$, $j_0\beta_0$ and $j_0\gamma_0$ as functions of j_0 for $z = 2$. Left: $n = 4$. As expected, all three functions are observed to have the same intercepts. Right: $n = 5$ with $r_L = 10^6$. In this case, convergence of the three functions to the same intercepts cannot be obtained numerically quite as accurately. In particular, the second intercept (i.e. magic value) is 20.3 for $j_0\alpha_0$ and $j_0\beta_0$, but 22.0 for $j_0\gamma_0$; the rest, however, are found to be the same.

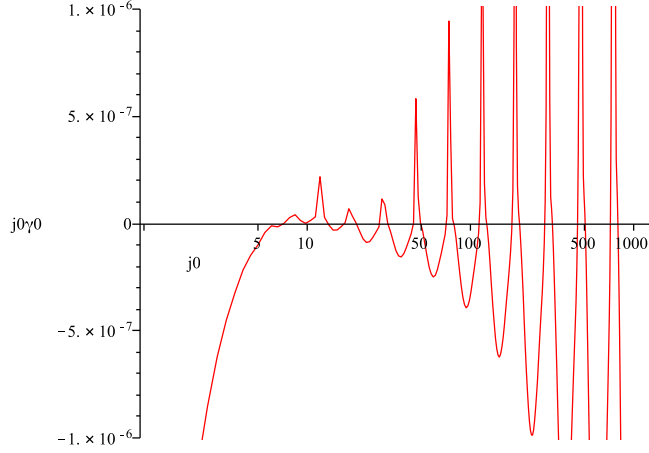


Figure 3b: Plot of $j_0 \gamma_0$ when $n = 6$. Numerical convergence of the intercepts of the above plot with those of $j_0 \alpha_0$ and $j_0 \beta_0$ is not very accurate in this case (becoming even worse for $n > 6$), and so here we base the list of magic values given in Figure 4 solely on the former.

n	Magic values for j_0 when $z = 2$
3	3.59, 21.8, 1.34×10^2 , ...
4	3.80, 18.0, 91.2, ...
5	4.60, 20.3, 1.01×10^2 , ...
6	7.20, 9.74, 13.5, ...

Figure 4: The three lowest magic values for $z = 2$ and $n = 3, 4, 5, 6$.

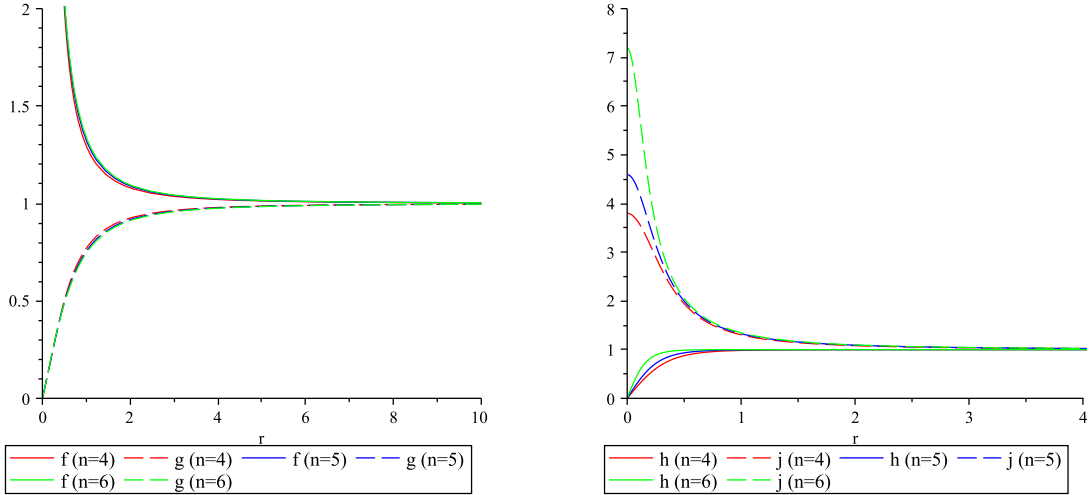


Figure 5a: The metric and gauge functions corresponding to the lowest magic value for $n = 4, 5, 6$. Left: The metric functions $f(r)$ and $g(r)$. Right: The gauge functions $h(r)$ and $j(r)$.

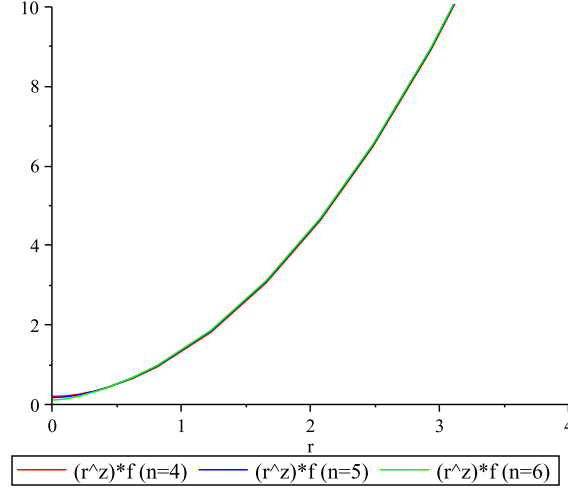


Figure 5b: The function $r^z f(r)$ for $n = 4, 5, 6$, illustrating that we indeed have soliton solutions.

3.3 Case III: $z > 2$

While (multiple) intercepts of the function $j_0 \gamma_0$ can still be obtained if z is made only *very slightly* greater than 2, none are seen to occur, in any dimension, once z becomes appreciably greater. In other words, we found no more magic values for $z > 2$. In particular, none exist for the zero modes (i.e. for $z = n - 1$) of any $n > 3$. In Figures 6a, 6b and 6c we plot $j_0 \gamma_0$ for, respectively, $n = 4, 5, 6$ and various $z > 2$. We find that magic values can still be obtained (i.e. intercepts of these graphs exist) only if, for instance, we have approximately $z < 2.021$ when $n = 4$, $z < 2.005$ when $n = 5$, and $z < 2.002$ when $n = 6$. The analogous plots of $j_0 \alpha_0$ and $j_0 \beta_0$ are found to be qualitatively similar, and so we have not included them here.

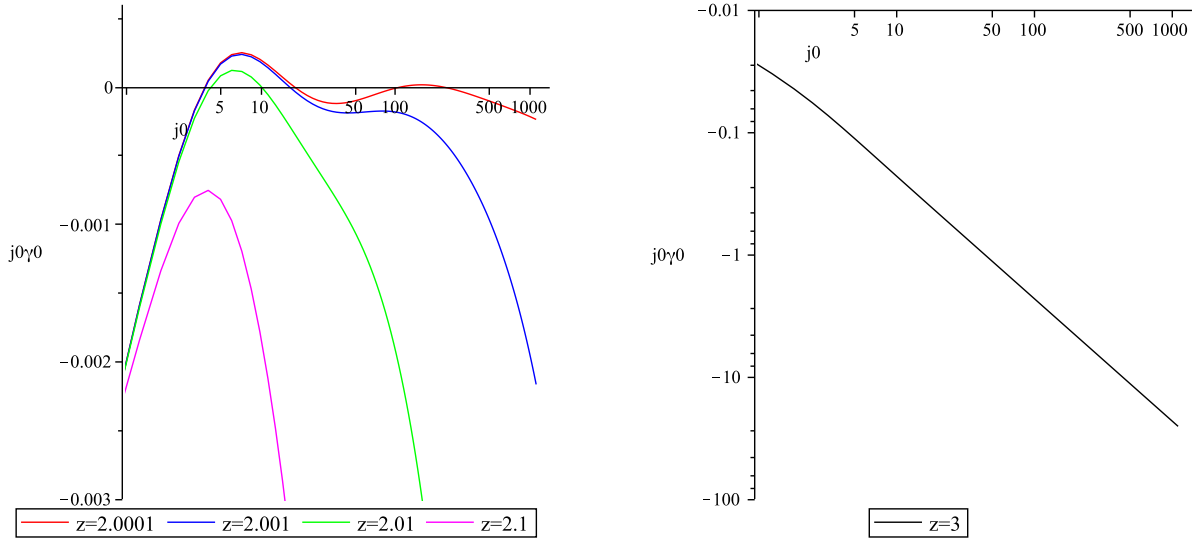


Figure 6a: Plots of $j_0 \gamma_0$ as a function of j_0 for $n = 4$ and various $z > 2$.

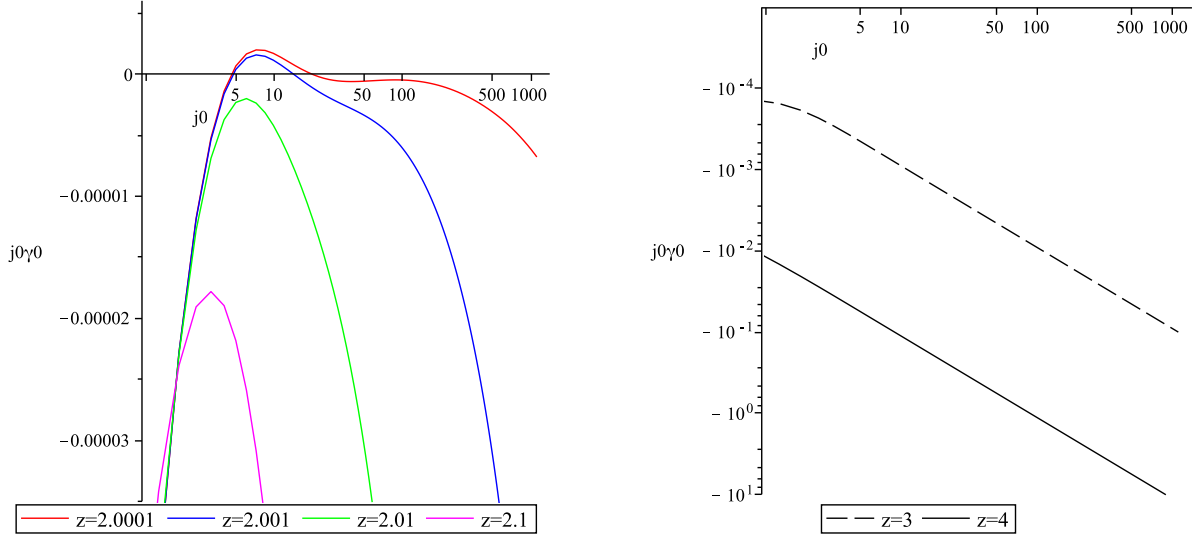


Figure 6b: Plots of $j_0 \gamma_0$ as a function of j_0 for $n = 5$ and various $z > 2$.

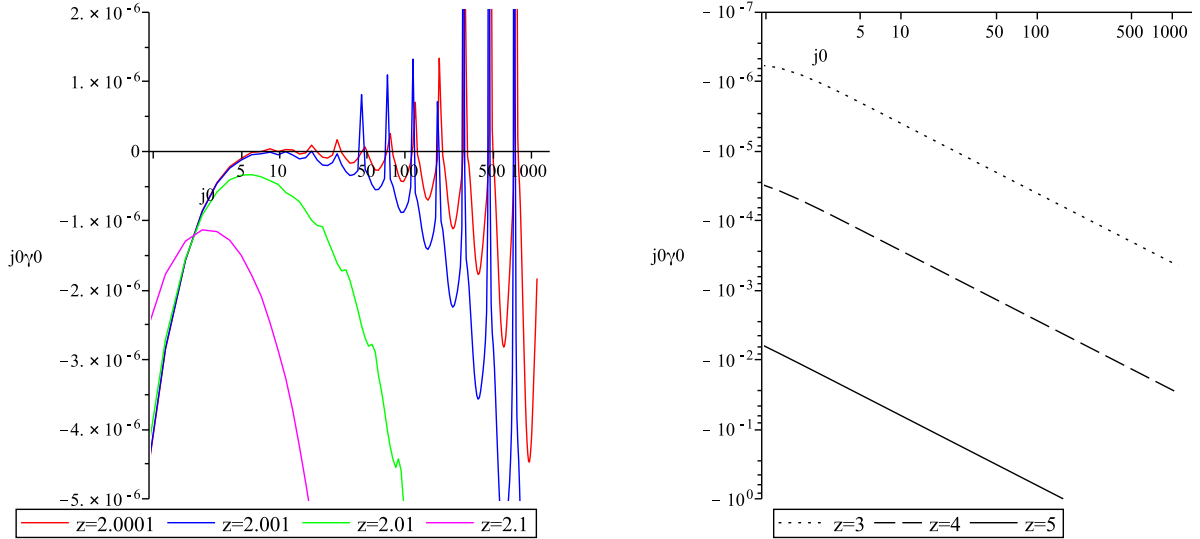


Figure 6c: Plots of $j_0 \gamma_0$ as a function of j_0 for $n = 6$ and various $z > 2$.

4 Conclusion

We have searched for soliton solutions in asymptotically Lifshitz spacetimes from $(3+1)$ [7] to $(n+1)$ dimensions. We have found that such solutions do exist, but with a somewhat surprising consequence: namely, solutions exist *only* when the critical exponent associated with the Lifshitz scaling is (very close to) 2, or smaller. In particular, $1 < z < 2$ and $z \approx 2$ (to within at most 0.021, $\forall n \geq 4$) yield, respectively, a single magic value and a discrete set of magic values, in any dimension. But, once z exceeds 2 sufficiently, no more magic values – and hence no more solutions for the metric and gauge functions – can be numerically found. This means, therefore, that no soliton solutions exist for the zero modes of any $n > 3$.

It would be interesting to understand in greater depth why $z = 2$ is such a special point in parameter space in all of the dimensions we investigated. Are such solutions stable, or will they undergo collapse to a black hole? The relationship between these solutions and the general (in)stability of asymptotically Lifshitz spacetimes [15] would be another interesting subject to investigate.

Acknowledgements

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Appendix

It can easily be verified that

$$f(r) = \frac{1}{rz} (f_0 + f_1 r^2 + f_2 r^4), \quad (17)$$

$$g(r) = r (g_0 + g_1 r^2 + g_2 r^4), \quad (18)$$

$$j(r) = j_0 + j_1 r^2 + j_2 r^4, \quad (19)$$

$$h(r) = r (h_0 + h_1 r^2 + h_2 r^4), \quad (20)$$

where

$$\begin{aligned} f_1 &= \frac{f_0}{2k\ell^2 n(n-1)} \left\{ \left[(n-1)^2 + z(n-2) + z^2 \right] + \left[(z-1)(n-1)^2 \right] j_0^2 \right\}, \\ f_2 &= \frac{-f_0}{8\ell^4 k^2 (n-1)^2 n^2 (n+2)} \left\{ \left[2(z-1)^4 + (z^2 + 2z - 7)(z-1)^2 n + 2(z^3 - z^2 - 3z + 4)n^2 \right. \right. \\ &\quad \left. \left. + (3z^2 - 4z - 2)n^3 + 2(z-1)n^4 + n^5 \right] + (z-1) \left[4(z^2 - 3z + 1) - 2(z^2 - 10z + 5)n - 4(z^2 + z - 1)n^2 \right. \right. \\ &\quad \left. \left. + 2(z-1)(z-4)n^3 + 8(z-1)n^4 - 2(z-1)n^5 \right] j_0^2 + (z-1)^2 [2 - 3n - 4n^2 + 10n^3 - 6n^4 + n^5] j_0^4 \right\}, \end{aligned}$$

$$g_0 = \frac{1}{k^{1/2}\ell},$$

$$g_1 = \frac{-1}{2k^{3/2}\ell^3 n(n-1)} \left\{ \left[n^2 + 1 - (2-z)(n+z) \right] - [(z-1)(n-1)] j_0^2 \right\},$$

$$\begin{aligned} g_2 &= \frac{-1}{8\ell^5 k^{5/2} (n-1)^2 n^2 (n+2)} \left\{ \left[-6(z-1)^4 - 3(z^2 + 2z - 7)(z-1)^2 n + (-6z^3 + 6z^2 + 18z - 24)n^2 \right. \right. \\ &\quad \left. \left. + (-9z^2 + 12z + 6)n^3 + 6(-z+1)n^4 - 3n^5 \right] + 2(z-1) \left[-2(3z^2 - 7z + 3) + (z^2 - 12z + 13)n \right. \right. \\ &\quad \left. \left. + (5z^2 - 9z - 3)n^2 + 9(z-1)n^3 + (-2z+5)n^4 \right] j_0^2 + (z-1)^2 \left[(n-1)^2 (4n^2 - 7n - 6) \right] j_0^4 \right\}, \end{aligned}$$

$$j_1 = \frac{-j_0}{2k\ell^2 n(n-1)} \left\{ \left[(n-1)^2 + z(3n - n^2 + z - 3) \right] + \left[(z-1)(n-1)^2 \right] j_0^2 \right\},$$

$$\begin{aligned} j_2 &= \frac{j_0}{8\ell^4 k^2 (n-1)^2 n^2 (n+2)} \left\{ \left[2(3z^2 - 8z + 3)(z-1)^2 + (3z^4 + 4z^3 - 41z^2 + 60z - 21)n \right. \right. \\ &\quad \left. \left. + (10z^3 - 14z^2 - 26z + 24)n^2 + (-4z^3 + 27z^2 - 20z - 6)n^3 + (-8z^2 + 18z - 6)n^4 + (z-1)(z-3)n^5 \right] \right. \\ &\quad \left. + 2(z-1) \left[2(3z^2 - 8z + 3) - (7z^2 - 31z + 19)n - 2(z^2 + 5z - 9)n^2 + z(3z - 13)n^3 + 2(5z - 4)n^4 \right. \right. \\ &\quad \left. \left. - 2(2z - 3)n^5 \right] j_0^2 + (z-1)^2 \left[(n-1)^3 (3n^2 - n - 6) \right] j_0^4 \right\}, \end{aligned}$$

$$\begin{aligned}
h_0 &= \frac{j_0 (n-1)}{k^{1/2} n \ell}, \\
h_1 &= \frac{-j_0}{2k^{3/2} \ell^3 n (n+2)} \left\{ [n^2 (2-z) + 4n (z-1) + 2z^2 - 5z + 2] + [(z-1) (n^2 - 3n + 2)] j_0^2 \right\}, \\
h_2 &= \frac{j_0}{8\ell^5 k^{5/2} (n-1) (n+4) n^2 (n+2)} \left\{ [8 (2z-1) (z-2) (z-1)^2 + (8z^4 + 10z^3 - 107z^2 + 154z - 56) n \right. \\
&\quad + (20z^3 - 18z^2 - 72z + 64) n^2 + (-6z^3 + 46z^2 - 36z - 16) n^3 + (-10z^2 + 32z - 16) n^4 + (z-2) (z-4) n^5] \\
&\quad + 4 (z-1) [4(2z-1)(z-2) - 2(3z^2 - 15z + 11) n - (4z^2 + 3z - 16) n^2 + (2z^2 - 13z + 4) n^3 + (7z - 8) n^4 \\
&\quad \left. + (-z + 2) n^5] j_0^2 + (z-1)^2 [(n-1)^2 (n-2) (3n^2 - 4n - 8)] j_0^4 \right\},
\end{aligned}$$

satisfy the four ODEs (12)-(15), at least up to sixth order in r . In Figure 7, we compare the above series solutions with the numerical results obtained (in Figure 5a) for the lowest magic value when $n = 4$, and find that they are in good agreement up to at least $r \approx 0.1$. These plots are found to be qualitatively similar for all other $n > 4$, and hence we refrain from showing them as well.

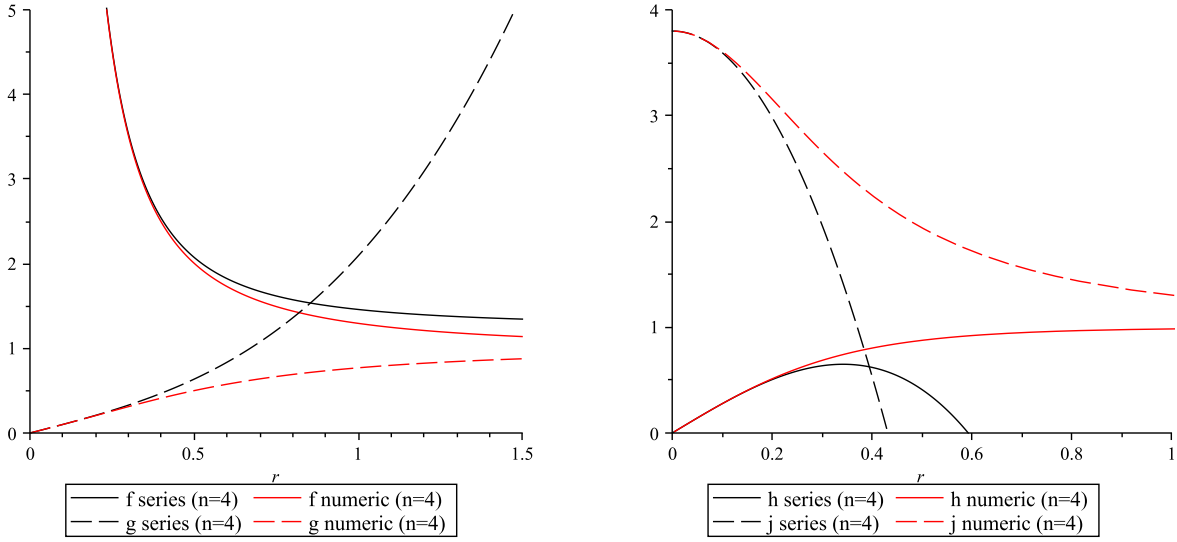


Figure 7: The metric and gauge functions corresponding to the lowest magic value for $n = 4$: numeric solutions (Figure 5a) versus series solutions ((17)-(20), truncated after two terms) for small r . Left: The metric functions $f(r)$ and $g(r)$. Right: The gauge functions $h(r)$ and $j(r)$.

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